## CMSC423: Bioinformatic Algorithms, Databases and Tools Lecture 8

Sequence alignment: inexact alignment<br>dynamic programming, gapped alignment

## Note

- Lecture 7 - suffix trees and suffix arrays will be rescheduled


## Exact alignment recap

- Exact matching can be done efficiently: O(|Text| + |Pattern|)
- Key idea: preprocess data to keep track of similar regions, then use information to "jump" over places where no match can occur



## Inexact matching: why?

- Redundancy in genetic code: nucleotide sequence may differ, but proteins the same
- Different amino-acid sequences still fold the same way: function unchanged (generally changing an amino-acid with a similar one doesn't affect protein function)
- Aligning ESTs (RNA sequences) to DNA need to account for gaps corresponding to exons
- Need to account for sequencing errors



## Several hemoglobins

HBB_HUMAN
HBB_HORSE
HBA_HUMAN
HBA_HORSE MYG_PHYCA GLB5_PETMA LGB2_LUPLU

FFESFGDLSTPDAVMGNPKVKAHGKKVL-----GAFSDGLAHLDNLKGTF FFDSFGDLSNPGAVMGNPKVKAHGKKVL-----HSFGEGVHHLDNLKGTF YFPHF-DLS-----HGSAQVKGHGKKVA-----DALTNAVAHVDDMPNAL YFPHF-DLS-----HGSAQVKAHGKKVG-----DALTLAVGHLDDLPGAL KFDRFKHLKTEAEMKASEDLKKHGVTVL-----TALGAILKKKGHHEAEL FFPKFKGLTTADQLKKSADVRWHAERII-----NAVNDAVASMDDTEKMS LFSFLKGTSEVP--QNNPELQAHAGKVFKLVYEAAIQLQVTGVVVTDATL . .: * : : :

## Warm-up - Longest Common Subsequence

- Given two strings of letters, identify longest string of letters that occurs, in the same order, in both strings

AG C GTAG
G C G A
GTCAG A

|  | $A$ | $G$ | $C$ | $G$ | $T$ | $A$ | $G$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $G$ |  | 1 |  | 1 |  |  | 1 |
| T |  |  |  |  | 1 |  |  |
| C |  |  | 1 |  |  |  |  |
| $A$ | 1 |  |  |  |  | 1 |  |
| G |  | 1 |  | 1 |  |  | 1 |
| A | 1 |  |  |  |  | 1 |  |

- Find the longest chain of 1 s , moving to the right and down


## Dynamic programming

- Idea: re-use previously computed information
- LCS[i,j] - longest common subsequence of strings S1[1..i], S2[1..j]
j

|  | A | G | G | T | A | G |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| G |  | 1 |  | 1 |  |  | 1 |
| T |  |  |  |  | 1 |  |  |
| C |  |  | 1 |  |  |  |  |
| A | 1 |  |  |  |  | 1 |  |
| G |  | 1 |  | 1 |  |  | 1 |
| A | 1 |  |  |  |  | 1 |  |
| Goal: find LCS[m,n] |  |  |  |  |  |  |  |

$\operatorname{LCS}[i, j]$ is the maximum of:
1.if S1[i] = S2[j]
$\operatorname{LCS}[i-1, j-1]+1$
else
$\operatorname{LCS}[i-1, j-1]$
2. $\operatorname{LCS}[i-1, j]$
3. $\operatorname{LCS}[i, j-1]$

## Computing the LCS table

Row 0 and column 0 easy to fill
Fill the rest column by column
Find the actual sequence: trace-back pointers

|  | $A$ | $G$ | $C$ | $G$ | $T$ | $A$ | $G$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $G$ | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| T | 0 | 1 |  |  |  |  |  |
| C | 0 | 1 |  |  |  |  |  |
| A | 1 | 1 |  |  |  |  |  |
| G | 0 | 2 |  |  |  |  |  |
| A | 1 | 2 |  |  |  |  |  |


|  | $A$ | $G$ | $C$ | $G$ | $T$ | $A$ | $G$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| G | 0 | 1 | 0 | 1 | 0 | 0 | 0 |  |
| T | 0 | $1:-1$ | -1 | 2 | 2 | -2 |  |  |
| C | 0 | -1 | 2 | -2 | 2 | $:$ | 2 | -2 |
| A | 1 | -1 | 2 | -2 | -2 | 3 | -3 |  |
| G | 0 | 2 | -2 | 3 | -3 | 3 | 4 |  |
| A | 1 | 2 | -2 | 3 | -3 | 4 | -4 |  |

Extending to sequence alignment
AG-C-GTAG
-GTCAG-A-

- In LCS, mis-alignments were free
- What happens if we pay for our "mistakes"? (this also allows us to account for "similar" aminoacids)
- Value[A, A] = 10
$-\operatorname{Value}[A, G]=-5$
- Value[A,-] = -2
- etc.
- The same dynamic programming algorithm works!


## The recurrences

$$
\begin{aligned}
& \text { AG-C-GTAG } \\
& \text {-GTCAG-A- }
\end{aligned}
$$

Score $[i, j]$ is the maximum of:

1. Score[i-1, j-1] + Value[S1[i],S2[j]]
AG-C-G
AG-C-G
-GTCAG
-GTCAT
2. Score[i - 1, j] + Value[S1[i], -] (S1[i] aligned to gap)

AG-C-GT
-GTCAG-
3. Score[i, j - 1] + Value[-, S2[j]] (S2[j] aligned to gap) AG-C--GTCA

## The dynamic programming table

Score[i,j] is the maximum of:

1. Score[i-1, j-1] + Value[S1[i-1],S2[j-1]] (S1[i-1], S2[j-1] aligned)
2. Score[i-1, j] + Value[S1[i], -] (S1[i] aligned to gap)
3. Score[i, j - 1] + Value[-, S2[j]] (S2[j] aligned to gap)

|  | - | A | G | C | G | T | A | G |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| - | 0 | -2 | -4 | -6 | -8 | -10 | -12 | -14 |
| G | -2 | -4 | -4 | 8 | 6 |  |  |  |
| T | -4 | -6 | 1 | -6 | -4 |  |  |  |
| C | -6 | -8 | 4 | 16 |  |  |  |  |
| A | -8 |  |  |  |  |  |  |  |
| G | -10 |  |  |  |  |  |  |  |
| A | -14 |  |  |  |  |  |  |  |

Value $(A, A)=10$
Value $(A, G)=-5$
Value ( $A,-$ ) = -2

Note: we only look at 3 adjacent boxes

## Local vs. global alignment

- Can we change the algorithm to allow S1 to be a substring of S2?

$$
\begin{aligned}
& \text { ACAGTTGACCCGTGCAT } \\
& ----\mathrm{TG}-\mathrm{CC}-\mathrm{G}------
\end{aligned}
$$

- Key idea: gaps at the end of S2 are free
- Simply change the first row in the DP table to 0s
- Answer is no longer Score[n, m], rather the largest value in the last row


## Sub-string alignment

|  | - | $A$ | $G$ | $C$ | $G$ | $T$ | $A$ | $G$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | -2 |  |  | 10 | 8 |  |  |  |
| G | -4 |  |  | 8 | 20 | 18 |  |  |
| T | -6 |  |  | 6 | 18 | 30 | 28 | 26 |

AgCGTAG
CGT

## Local alignment

- What if we just want a region of similarity?

$$
\begin{aligned}
& \text { ACAGTTGACCCGTGCAT } \\
& \text { |। ।। । } \\
& \text { GTCATG-CC-GAGATCG }
\end{aligned}
$$

- First row and column set to 0s
- Allow alignment to start anywhere:

Score[i,j] $=\max \{0$, case 1 , case 2 , case 3$\}$

- Answer is location in matrix with highest score


## Local alignment

|  |  | $A$ | $G$ | $C$ | $G$ | $T$ | $A$ | $G$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 |  |  |  |  |  |  |  |
| T | 0 |  | 0 |  |  |  |  |  |
| C | 0 |  |  | 10 |  |  |  |  |
| G | 0 |  |  |  | 20 |  |  |  |
| T | 0 |  |  |  |  | 30 |  |  |
| C | 0 | AGCGTAG <br> III <br> CTCGTC |  |  |  |  |  |  |

## Various flavors of alignment

- Alignment problem also called "edit distance" - how many changes do you have to make to a string to convert it into another one.
- Edit distance also called Levenshtein distance
- Local alignment - Smith-Waterman
- Global alignment - Needleman-Wunsch

