# CMSC423: Bioinformatic Algorithms, Databases and Tools Lecture 9 

Sequence alignment: inexact alignment
dynamic programming, gapped alignment, heuristics

## Play around with alignments

- USC alignment library (seqaln)
http://www.mhoenicka.de/software/cygwinports/seqaln.html


## Global alignment recap

Score[i,j] is the maximum of:

1. Score[i-1, j-1] + Value[S1[i-1],S2[j-1]] (S1[i-1], S2[j-1] aligned)
2. Score[i-1, j] + Value[S1[i], -] (S1[i] aligned to gap)
3. Score[i, j-1] + Value[-, S2[j]] (S2[j] aligned to gap)

|  | - | $A$ | G | C | G | T | A | G |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| - |  |  |  |  |  |  |  |  |
| G |  |  |  |  |  |  |  |  |
| $T$ |  |  |  |  |  |  |  |  |
| C |  |  |  |  |  |  |  |  |
| A |  |  |  |  |  |  |  |  |
| G |  |  |  |  |  |  |  |  |
| A |  |  |  |  |  |  |  |  |
| C |  |  |  |  |  |  |  |  |

$\operatorname{Value}(A, A)=10$ Value $(A, G)=-5$
Value $(A,-)=-2$

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3. Score[i, j-1] + Value[-, S2[j]] (S2[j] aligned to gap)

|  | - | $A$ | $G$ | $C$ | $G$ | $T$ | $A$ | $G$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| - | 0 | -4 | -8 | -12 | -16 | -20 | -24 | -28 |
| G | -4 | -5 | 6 | 2 | -2 | -6 | -10 | -14 |
| T | -8 | -9 | 2 | 1 | -3 | 8 | 4 | 0 |
| C | -12 | -13 | -2 | 12 | 8 | 4 | 3 | -1 |
| A | -16 | -2 | -6 | 8 | 7 | 3 | 14 | 10 |
| G | -20 | -6 | 8 | 4 | 18 | 14 | 10 | 24 |
| $A$ | -24 | -10 | 4 | 3 | 14 | 13 | 24 | 20 |
| C | -28 | -14 | 0 | 14 | 10 | 9 | 20 | 19 |

AG-C-GTAG
-GTCAG-AC
$\operatorname{Value}(A, A)=10$
Value $(A, G)=-5$
Value $(A,-)=-4$

## Local alignment recap

Score[i,j] is the maximum of:
0. 0

1. Score[i-1, j-1] + Value[S1[i-1],S2[j-1]] (S1[i-1], S2[j-1] aligned)
2. Score[i-1, j] + Value[S1[i], -] (S1[i] aligned to gap)
3. Score[i, j - 1] + Value[-, S2[j]] (S2[j] aligned to gap)

|  | - | A | G | C | G | T | A | G |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| - |  |  |  |  |  |  |  |  |
| G |  |  |  |  |  |  |  |  |
| T |  |  |  |  |  |  |  |  |
| C |  |  |  |  |  |  |  |  |
| A |  |  |  |  |  |  |  |  |
| G |  |  |  |  |  |  |  |  |
| A |  |  |  |  |  |  |  |  |
| C |  |  |  |  |  |  |  |  |

AGCGTAG
GTCAGAC

## How much do we pay for gaps?

- In the edit-distance/alignment framework Cost(n gaps in a row) $=$ n * Cost(gap)
- This doesn't work for e.g. RNA-DNA alignments ACAGTTCGACTAGAGGACCTAGACCACTCTGT

TTCGA-----------TAGACCAC

- Affine gap penalties

Cost(n gaps in a row) $=\operatorname{Cost}($ gap open $)+\mathrm{n}$ * $\operatorname{Cost(gap)~}$

- Gap opening penalty is high, gap extension penalty is low (once we start a gap we might as well pile more gaps on top)


## Dynamic programming solution

- Traditional 1-table approach doesn't work anymore
- Instead, use 4 tables:
- V - stores value of best alignment between S1[1..i], S2[1..j]
- G - best alignment between S1[1..i], S2[1..j] s.t. S1[i] aligned with S2[j]
- E - best alignment between S1[1..i], S2[1..j], s.t. alignment ends with gap in S1
- F - best alignment between S1[1..i], S2[1..j], s.t. alignment ends with gap in S2
- $V[i, j]=\max (E[i, j], F[i, j], G[i, j])$
- As in traditional approach, find box in V matrix where $\mathrm{V}[\mathrm{i}, \mathrm{j}]$ is maximal.

[^0]
## Running times

- All these algorithms run in $\mathrm{O}(\mathrm{mn})$ - quadratic time
- Note - this is significantly worse than exact matching
- On Wednesday we'll talk about speed-up opportunities
- BTW, how much space is needed?
- If we only need to find the best score (not the exact alignment as well) - $\mathrm{O}(\min (\mathrm{m}, \mathrm{n}))$
- If we need to find the best alignment - elegant divide and conquer algorithm leads to linear space solution.


## Where do the alignment scores come from?

- PAM matrices
- PAM1 - based on frequency of mutations between closely related proteins (within 1 "evolutionary step")
- PAM 2 - ... within 2 evolutionary steps
- ... PAM 250 - commonly used
- BLOSUM matrices
- Frequency of mutations between proteins that are $\mathrm{x} \%$ similar
- BLOSUM100 - based on proteins that are exactly the same (e.g. $\operatorname{score}(A, A)$ is defined but not score $(A, G)$ )
- BLOSUM62 - commonly used
- gap scores usually determined empirically


## BLOSUM62



## Heuristics

- What if limit the \# of differences allowed? E.g. we expect the sequences to be very similar.
- Compute 'banded' alignment - stay within \# of differences ( $k$ ) from the diagonal.
- Optimal alignment cannot stray too far from diagonal

$\mathrm{O}(\mathrm{km})$ running time and space
- What if we do not know k? Do binary search to find it


## Exclusion methods

- Assume P must match T with at most k errors. Find places in $T$ where $P$ cannot match.
- Split $P$ into floor( $\mathbf{n} / \mathbf{k}+1$ )-sized chunks.
- If $P$ matches $T$ with less than $k$ errors => at least one chunk matches with no errors
- Use any exact matching algorithm to find places where a chunk matches $T$, then run dynamic programming in that vicinity.
- Running time, on average $\mathrm{O}(\mathrm{m})$


## Exclusion methods



## "Famous" approaches

- FASTA (Pearson et al.)
- Take all k-mers (substrings of length k) from Pattern and identify whether and where they match in the Text
- Assume the k-mer starting at pos'n i in Pattern matches at position j in Text, remember $(\mathrm{j}-\mathrm{i})$ - the diagonal on which the match occured
- Identify "heavy" diagonals - diagonals where many k-mers match, then refine the diagonals with Smith Waterman
- Also look for off-diagonal matches to account for gaps



## "Famous" approaches

- BLAST (Altschul et al.)
- Find short k-mer matches
- Also search for possible inexact matches, e.g. all k-mers within 1 difference from current one.
- Extend exact matches with Smith-Waterman algorithm
- Assign probabilistic scores to matches: what is the probability of finding a match with the same S-W alignment score just by chance (e.g. matching a random string)?


[^0]:    Affine gap recurrences

    - $V[i, j]=\max [E[i, j], F[i, j], G[i, j]]$
    - $G[i, j]=V[i-1, j-1]+\operatorname{Value}(S 1[i], S 2[j])$
    - irrespective how we got here (hence use of V), S1[i] and S2[j] are matched
    - $E[i, j]=\max \{E[i, j-1], V[i, j-1]-$ GapOpen $\}-$ GapExtend
    - either we add a gap in S1 to an existing one (E-GapExtend)
    - or we add a gap in S1 when there was none (V-GapOpenGapExtend)
    - $F[i, j]=\max \{F[i-1, j], \mathrm{V}[i-1, j]-$ GapOpen $\}-$ GapExtend - either we add a gap in S2 to an existing one (F-GapExtend)
    - or we add a gap in S2 when there was none (V-GapOpenGapExtend)

