

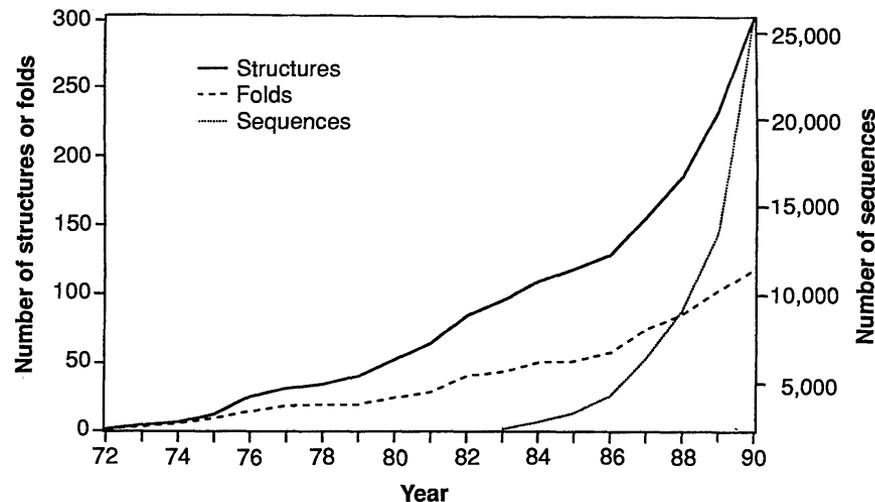
CMSC 858P Lecture 26: Protein  
folding – threading  
5/6/08

# Glossary

- Residue – any single amino-acid
- Side-chain – chemical group off the backbone
- Peptide – a short chunk of protein
- Polypeptide – protein

# Threading: reverse structure prediction

- Main hypothesis: while there are many protein sequences, there are much fewer folds. I.e. nature keeps reinventing useful structures



- Given a database of structures and a query string, find which structure “fits” the string best

# Initial idea: 3D-1D scores

- From a 3D structure, determine “environment” for every amino-acid
  - buried (inside the protein)
  - outside
  - inner side of helix
  - outer side of helix
  - etc...
- Annotate each position in protein with the environment information  
ACKCAHGT -> E<sub>1</sub>E<sub>2</sub>E<sub>1</sub>E<sub>3</sub>E<sub>4</sub>E<sub>2</sub>E<sub>3</sub>E<sub>1</sub>E<sub>4</sub>
- Why this is reasonable? Amino-acids have “preference” for specific environments

# Alignment to an environment string

- Idea: use gapped alignment algorithm to estimate how likely it is for a sequence to conform to a structure (represented as an environment string)

- $$\begin{array}{cccccccc} E_1 E_2 - & E_1 E_3 - & - & E_4 E_2 - & E_3 E_1 E_4 \\ A G H - & K T G A L & K M N G \end{array}$$

- Question: what is the score of aligning an amino-acid to an environment?

# Answer: use statistics

- For each environment – calculate likelihood (observed frequency) of all amino-acids based on known structures
- For each environment – empirical estimation of gap opening/extension penalties
- Alignment algorithm – use Gribskov's profile method: replace each environment character with the amino-acid frequency table for that environment

$E_1$

A 0.22

K 0.15

W 0.08

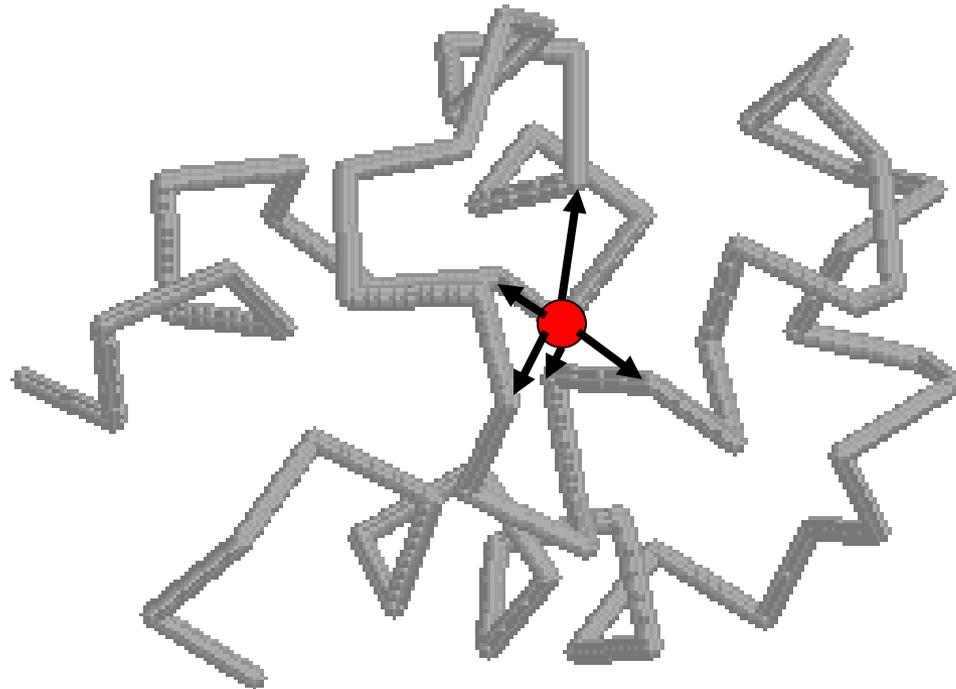
...

$$S(E_1, G) = \sum_{AA} S(AA, G) * \text{freq}_{E_1}(AA)$$

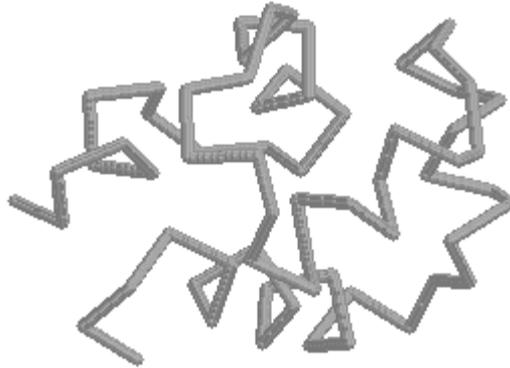
$S(AA, G)$  – e.g. from BLOSUM matrix

# Environments – not good enough

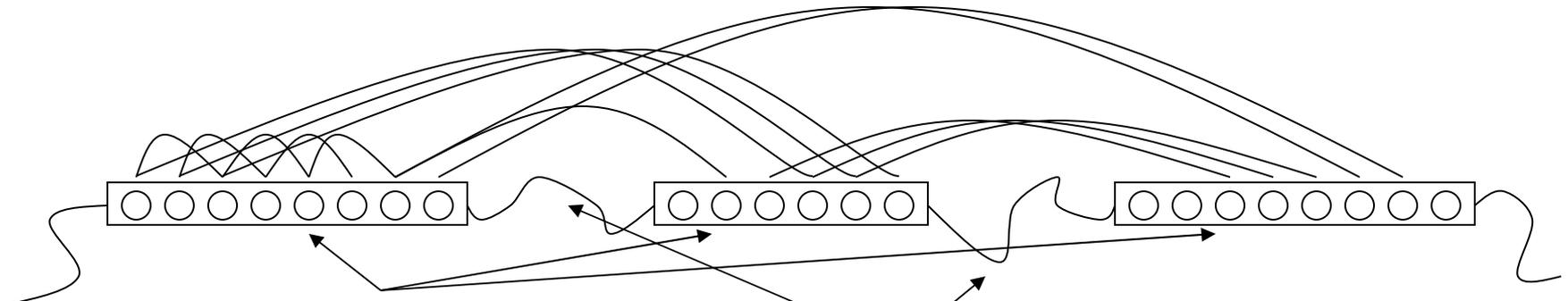
- Each amino-acid may have multiple contacts



# A better model



residue interactions (and associated energy parameters)



core "modules" (helix, sheet, etc.)

variable length connections (gaps)

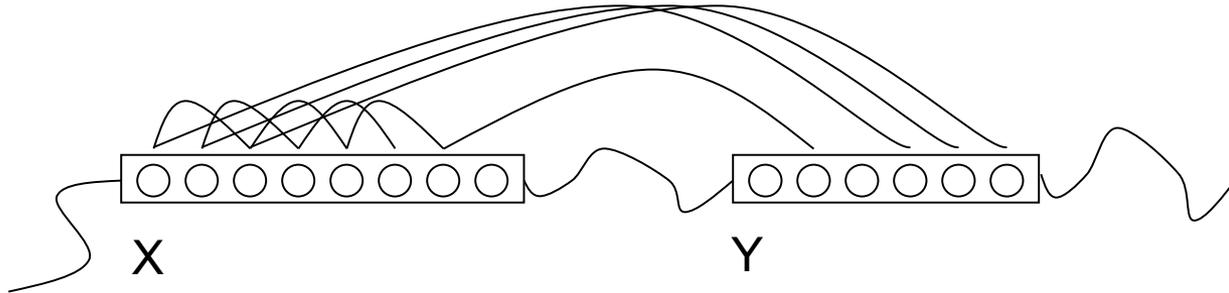
# The threading problem

- Model assumptions:
  - loop AA composition and length contributes to energy score (note: can also place restrictions on minimum/maximum size in gaps)
  - interactions are pair-wise: interaction energy depends on at most two AAs
  - individual AAs in core modules also contribute to energy due to local environment
- Thread a protein sequence through a structure model s.t.
  - the place-holders are filled with amino-acids
  - a variable number of amino-acids fall in the gaps
  - overall energy is minimized
- Easy to say, hard to do: Thus defined (variable length gaps AND pair-wise interactions) the problem is NP-hard!

# NP-hard => heuristics

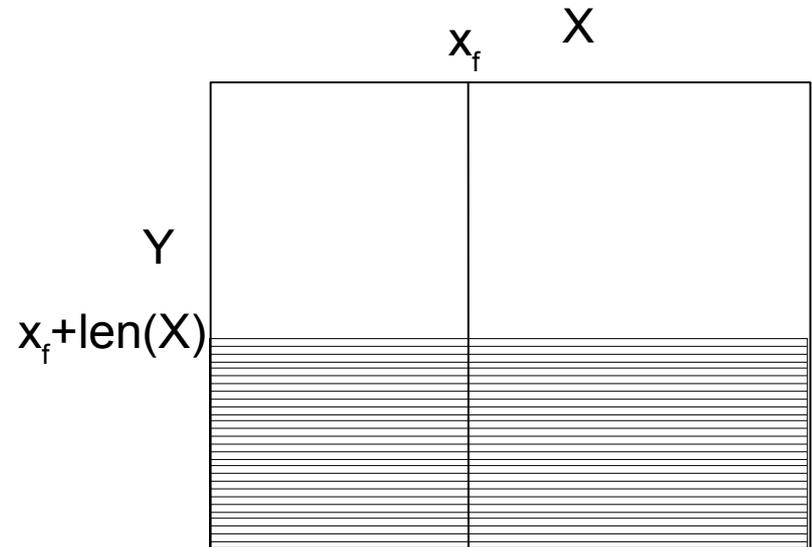
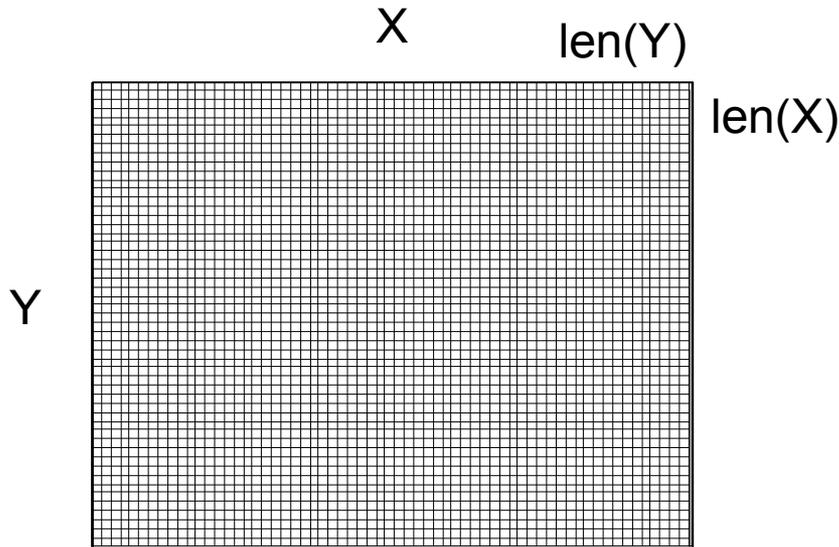
- Branch and bound (Lathrop, Smith)
  - Represent all possible folds (search space) s.t. it is easy to compute a lower bound on the score
  - Note: a threading is uniquely defined by the coordinates of the core elements – a set of threadings is a hyper-rectangle in a C-dimensional space where C is the # of core elements
  - Divide search space and compute energy lower-bounds on each sub-division (choose a dimension (core) and a coordinate and split hyper-rectangle at that location)
  - Recurse on sub-division with lowest lower-bound

# Hyper-rectangle heuristic

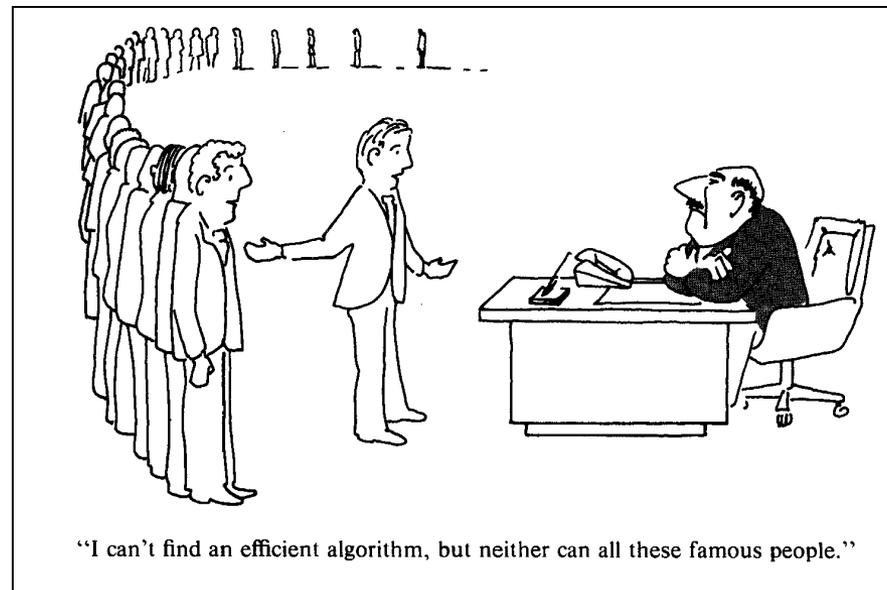
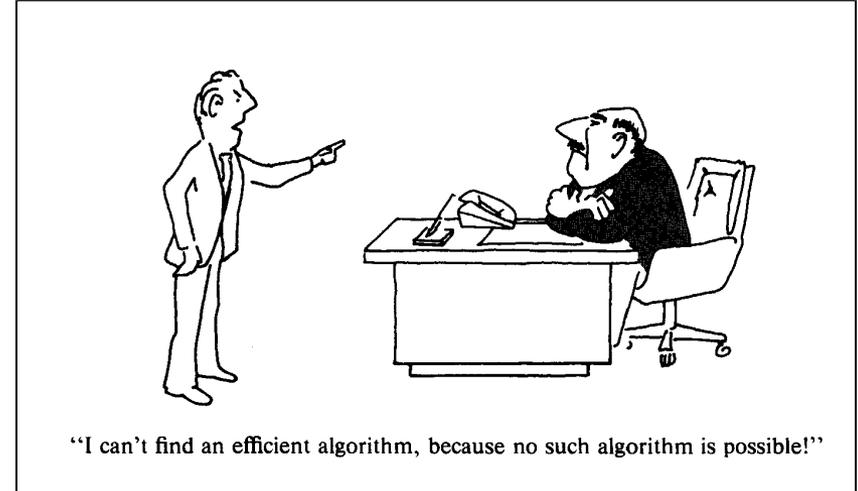
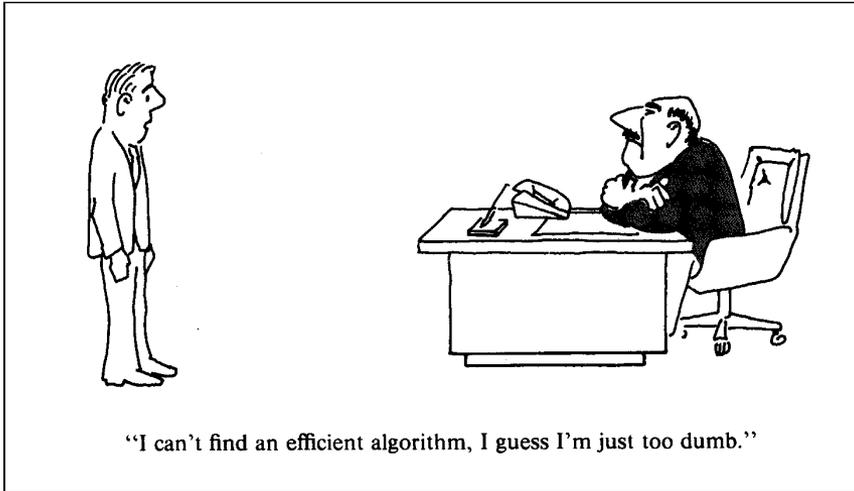


Each “module” corresponds to a dimension - offset of module in the protein

Fixing one module restricts the flexibility in assigning the remaining modules (imagine beads on a string)



# NP-completeness



From: **Computers  
and  
Intractability**

M. R. Garey and  
D. S. Johnson  
(*W. H. Freeman*  
1979)

# Threading is NP-hard - proof

- Reduction from ONE-IN-THREE 3 SAT
  - $n$  boolean variables,  $k$  boolean clauses with exactly 3 literals
  - 3 SAT – is there a setting of the variables such that all clauses are simultaneously true?
  - ONE-IN-THREE 3SAT – 3SAT but each clause made true by exactly one literal
- Proof: for any instance of 3SAT, create an instance of the protein threading problem s.t. a solution to the threading problem implies a solution to 3SAT

# Proof ...cont

- Protein sequence
  - T, F – state of each boolean value
  - P,Q,R – which literal makes a clause true
  - protein: PQR PQR PQR...TFTFTF....
- Core model
  - one core element (with one AA) for each clause
  - one core element (with one AA) for each boolean
  - interactions from each clause to the booleans present in it. edge also encodes which literal (1,2,3) and whether value is negated
  - edge score = 0 if label consistent with amino-acid assignment and 1 otherwise (e.g. QF is consistent with edge 2,NOT)
  - optimal threading has score 0 and solves 3SAT

# Discussion

- Both variable length gaps and pairwise interactions are essential!
- If no variable length gaps – can try all threadings in polynomial time irrespective of interactions
- If no pairwise interactions – dynamic programming can figure out the correct assignment (essentially the alignment problem)

# Structure to structure alignment

- Given two proteins with known structure, how do we align them to each other?
- Double Dynamic Programming
  - distance matrix depends on distance between residues
  - pick a pair of residues  $(i,j)$  and assume they are paired up
  - use dynamic programming to align the rest of the protein – score will represent score for pairing of  $i,j$
  - use a final dynamic programming step to align the proteins based on scores determined above

# Example

