CMSC 423 Homework #4:

Due: Dec. 14 at midnight

You may discuss these problems with other students in this class, but you **must write up your solutions independently**, without using common notes or worksheets. You must indicate at the top of your homework who you worked with. Your write up should be clear, concise, and neat. You are trying to convince a skeptical reader that your algorithms or answers are correct. Messy or hard-to-read homeworks will not be graded.

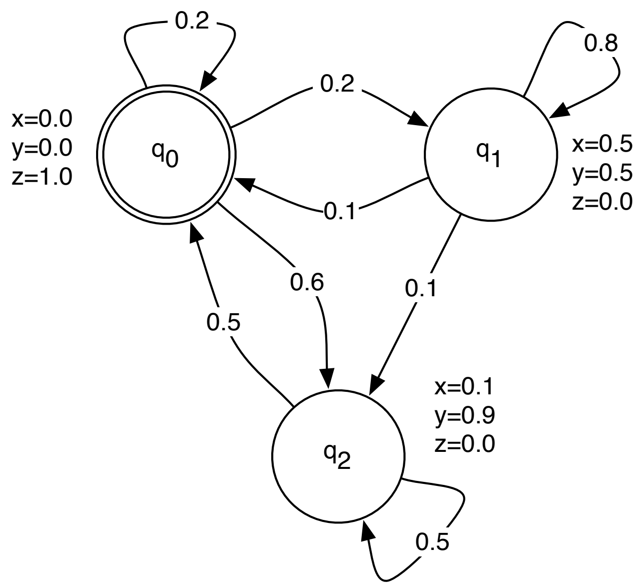
1. Suppose Alice is has two coins, one that is fair (probably heads = probability tails = 0.5) and one that is biased (probability heads = 0.25, probabilities of tails = 0.75). She chooses one of the coins (fair and biased are chosen with equal probability), and starts flipping the coin. Subject to the constraints below, she has a 0.1 probability of switching which coin she uses and 0.9 of continuing to use the same coin.

   (a) Suppose to avoid detection Alice always makes **at least** 5 flips with a coin before switching. Draw an HMM that can model this situation.

   (b) Suppose instead of requiring ≥ 5 flips before switching, we require Alice flips a given coin **no more than** k times before switching, where k is some parameter $\leq n$.

   Give an algorithm that, given k and a sequence x of n results of coin flips (H or T), finds the most probable sequence of fair/biased coin usages under this restriction. In other words, modify the Viterbi algorithm to find the best path that never stays in any state for longer than k consecutive steps. The output should be a list of n instances of "Fair" and "Biased". Your algorithm should use $O(n)$ space and $O(nk)$ time.

2. Consider the following hidden Markov model:



   Assume we start in state q_0.

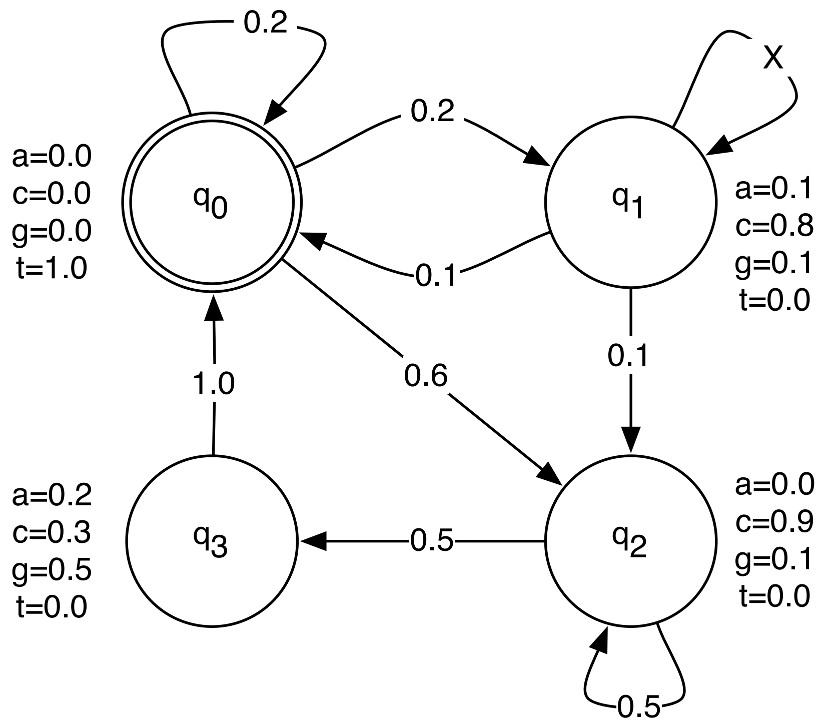
   (a) What is the probability of the path $q_0 \to q_1 \to q_2 \to q_0$?

   (b) What is the probability that `zyyy` was emitted if the path taken was $q_0 \to q_1 \to q_1 \to q_2$?

   (c) Fill in the missing entries of the following Viterbi dynamic programming matrix for the string `zxyx` and the above HMM. Show the backtracing arrows for the entries you fill in.

q2	0.000000	0.020000	________	0.000450
q1	0.000000	0.033333	________	0.005333
q0	0.333333	0.000000	________	0.000000
	z	x	y	x

3. Consider the following HMM, where transition probabilities are on the edges and emission probabilities are given in tables next to the nodes:



(a) What must the transition probability X adjacent to state q_1 be?

(b) Suppose we start in state q_0. Give two paths that could emit the string `tagcat`. What are their probabilities?

(c) Suppose we start in state q_0 with probability 1.0. Compute and show the Viterbi dynamic programming matrix for the string `tacccgt`. What is the highest probability path for this string?